Embeddings Learned by Gradient Descent

Hinrich Schütze

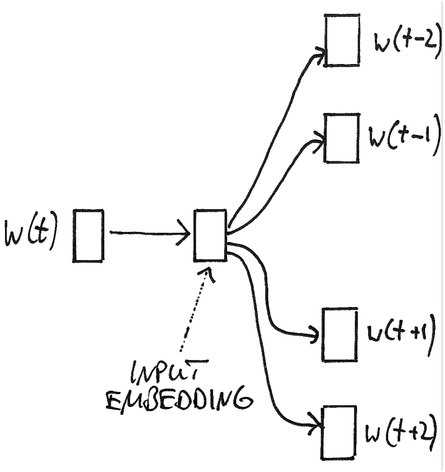
Center for Information and Language Processing, LMU Munich

2017-07-20

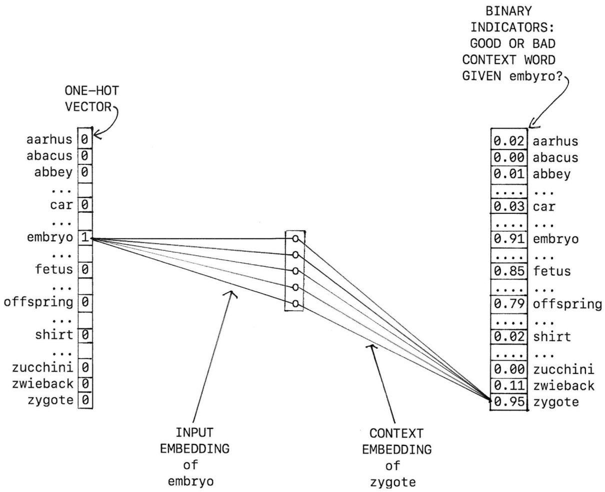
Overview

Outline

word2vec skipgram
predict, based on input word, a context word



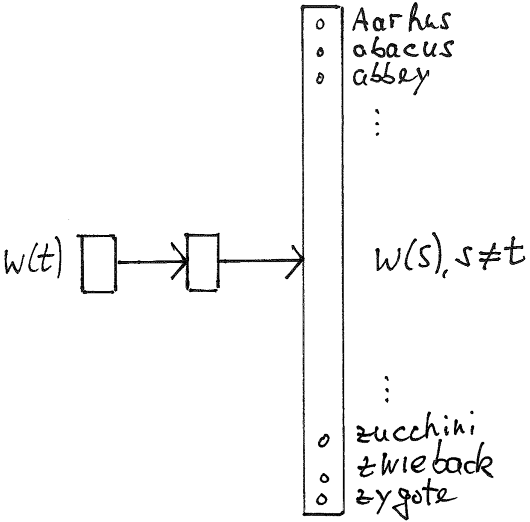
word2vec skipgram
predict, based on input word, a context word



Three versions of word2vec skipgram

- All three share skipgram objective (previous slide): predict, based on input word, a context word
- 1. Matrix factorization (SVD) of PPMI matrix
  - Tuesday's lecture
- 2. skipgram negative sampling (SGNS) using GD
  - Today's topic
  - Levy&Goldberg show rough equivalence: SGNS $\approx$ SVD-of-PPMI-matrix
  - No rigorous proof?
- 3. hierarchical softmax (skipgram HS)
  - skipgram HS vs. SGNS: different objectives

skipgram softmax



skipgram softmax: objective

$$\arg\max_{\theta} \sum_{(w,c)\in D} \log \frac{\exp(\vec{v}_w \cdot \vec{v}_c)}{\sum_{c' \in V} \exp(\vec{v}_w \cdot \vec{v}_{c'})}$$

(hierarchical softmax is hierarchical version of this)

Three versions of skipgram: Learning algorithms

w2v skipgram SGNS (original)	gradient descent
w2v skipgram SGNS (Levy&Goldberg)	SVD
w2v skipgram hierarchical softmax	gradient descent

Outline

skipgram negative sampling (SGNS): objective

skipgram negative sampling (SGNS): objective (not!)

$$\arg\max_{\theta}\Big[\sum_{(w,c)\in D}(\vec{v}_w\cdot\vec{v}_c)+\beta\sum_{(w,c)\in V\times V}(-\vec{v}_w\cdot\vec{v}_c)\Big]$$

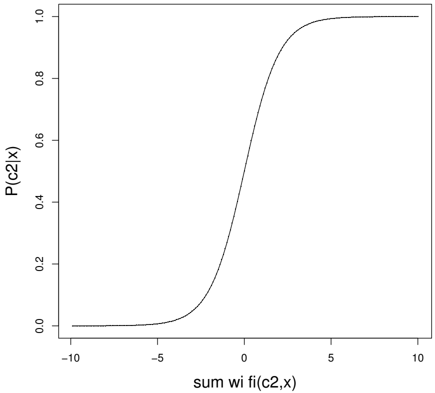
- Training set D: set of word-context pairs (w, c)
- We learn an embedding $\vec{v}_w$ for each w.
- We learn an embedding $\vec{v}_c$ for each c.
- Note that each word has two embeddings: an input embedding and a context embedding
- We generally only use the input embedding.
- make dot product of “true” pairs as big as possible
- dot product of “false” pairs as small as possible

skipgram negative sampling (SGNS): objective

$$\arg\max_{\theta}[\sum_{(w,c)\in D} \log \sigma(\vec{v}_w \cdot \vec{v}_c) + \beta \sum_{(w,c)\in V\times V} \log \sigma(-\vec{v}_w \cdot \vec{v}_c)]$$

- $\sigma(x) = 1/(1 + e^{-x})$
- Training set D: set of word-context pairs (w, c)
- We learn an embedding $\vec{v}_w$ for each w.
- We learn an embedding $\vec{v}_c$ for each c.
- Note that each word has two embeddings: an input embedding and a context embedding
- We generally only use the input embedding.
- make dot product of “true” pairs as big as possible
- dot product of “false” pairs as small as possible

σ: Logistic = Sigmoid



Housing prices in Portland

input variable **x** size (feet^2)	output variable **y** price (\$) in 1000s
2104	460
1416	232
1534	315
852	178

We will use m for the number of training examples.

Setup to learn housing price predictor using GD
Next: Setup for word2vec skipgram

- Hypothesis:

$$h_\theta = \theta_0 + \theta_1 x$$

- Parameters:

$$\theta = (\theta_0, \theta_1)$$

- Cost function:

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2$$

- Objective: $\text{minimize}_{\theta_0, \theta_1} J(\theta_0, \theta_1)$

Parameters

house prices: $\theta = (\theta_0, \theta_1)$

dimensionality of embeddings: d, size of vocabulary: n, word embeddings θ, context embeddings η

word2vec skipgram:

$\theta_{11}, \theta_{12}, \ldots, \theta_{1d}$

$\theta_{21}, \theta_{22}, \ldots, \theta_{2d}$

...

$\theta_{n1}, \theta_{n2}, \ldots, \theta_{nd}$

$\eta_{11}, \eta_{12}, \ldots, \eta_{1d}$

$\eta_{21}, \eta_{22}, \ldots, \eta_{2d}$

...

$\eta_{n1}, \eta_{n2}, \ldots, \eta_{nd}$

Hypothesis

house prices: $h_\theta = \theta_0 + \theta_1 x$

word2vec skipgram:

$$h_{\theta,\eta}(i) = \theta_i \qquad (= \eta_j)$$

Cost function

house prices:

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2$$

word2vec skipgram It's a reward function!

$$[\sum_{(w,c)\in D} \log \sigma(\vec{v}_w \cdot \vec{v}_c) + \beta \sum_{(w,c)\in V\times V} \log \sigma(-\vec{v}_w \cdot \vec{v}_c)]$$

$$J(\theta, \eta) = [\sum_{(w,c)\in D} \log \sigma(\theta(w) \cdot \eta(c)) + \beta \sum_{(w,c)\in V\times V} \log \sigma(-\theta(w) \cdot \eta(c))]$$

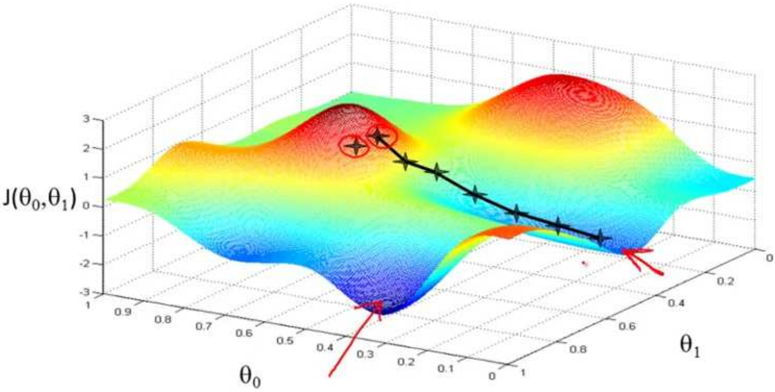
Objective

house prices: gradient descent

$$\text{minimize}_{\theta_0,\theta_1} J(\theta_0, \theta_1)$$

word2vec skipgram: gradient ascent

$$\text{maximize}_{\theta,\eta} J(\theta, \eta)$$


J(θ0,θ1)
θ0
θ1
3
2
1
0
-1
-2
-3
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1

Exercise

- What is the maximum value that the objective can take in word2vec skipgram? (focus on first term, below)
- Are we likely to find parameters for which we reach the maximum? (focus on first term, below)
- (Recall: $\sigma(x) = 1/(1 + e^{-x})$)
- Why?

$$\arg\max_{\theta} \sum_{(w,c) \in D} \log \sigma(\theta(w) \cdot \eta(c))$$

house prices	word2vec skipgram
θ_0, θ_1	$2\|V\|d$ parameters: θ, η
$h_\theta(x) = \theta_0 + \theta_1 x$	$h_{\theta,\eta}(i) = \theta(i) \approx \eta(c)$
$J(\theta) =$	$J(\theta, \eta) =$
$1/(2m) \sum (h_\theta(x^{(i)}) - y^{(i)})^2$	$\sum_{(w,c) \in D} \log \sigma(\theta(w) \cdot \eta(c))$
	$+\beta \sum_{(w,c) \in V \times V} \log \sigma(-\theta(w) \cdot \eta(c))$
$\text{argmin}_\theta J(\theta)$	$\text{argmax}_{\theta,\eta} J(\theta, \eta)$

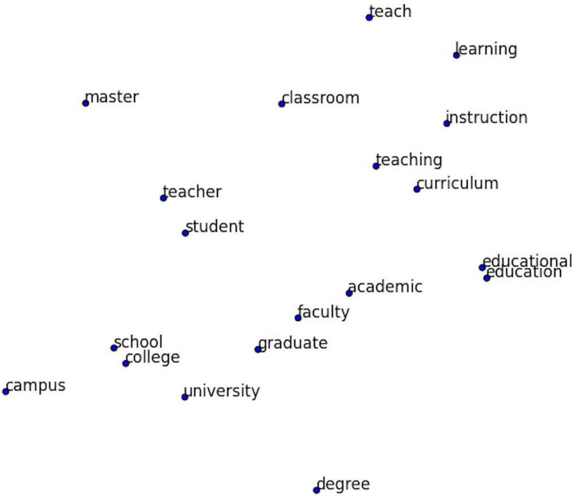
Outline

TensorBoard

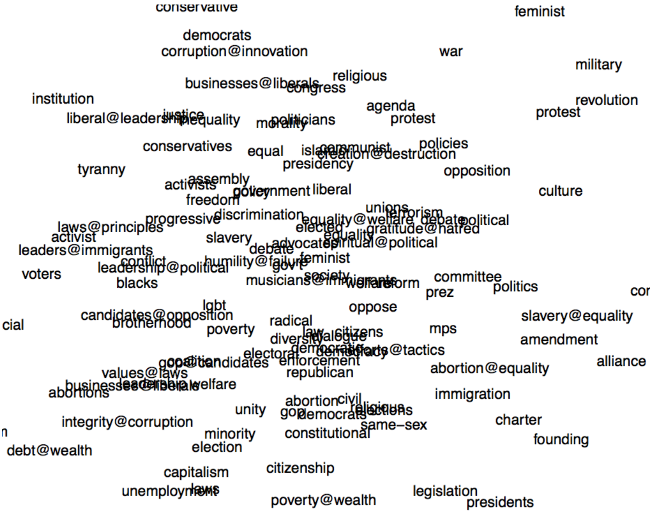
Visualization

- How to understand / analyze embeddings?
- Frequently used: two-dimensional projections
- Methods / software
  - Traditional:
    multidimensional scaling, PCA
  - t-SNE
    `https://lvdmaaten.github.io/tsne/`
  - gensim
    `https://radimrehurek.com/gensim/`
  - Pretty much all methods are implemented in R:
    `https://www.r-project.org`
- Important: The two dimensions are not interpretable.

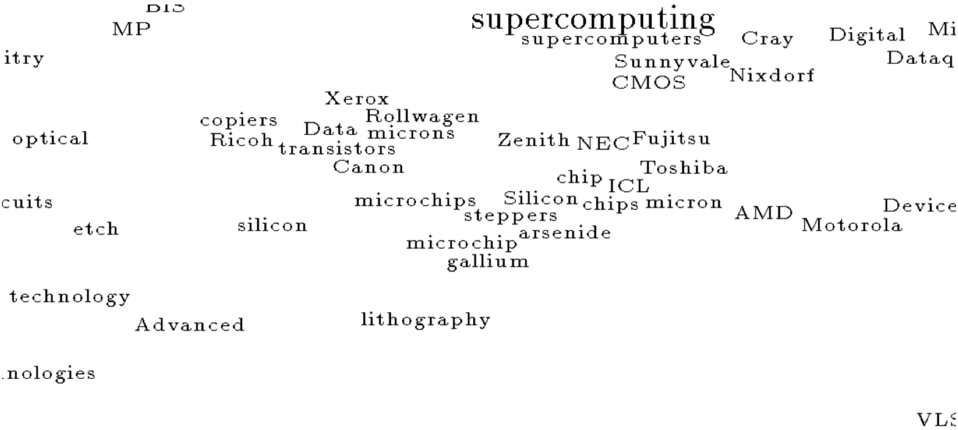
2D projection of embeddings



2D projection of embeddings

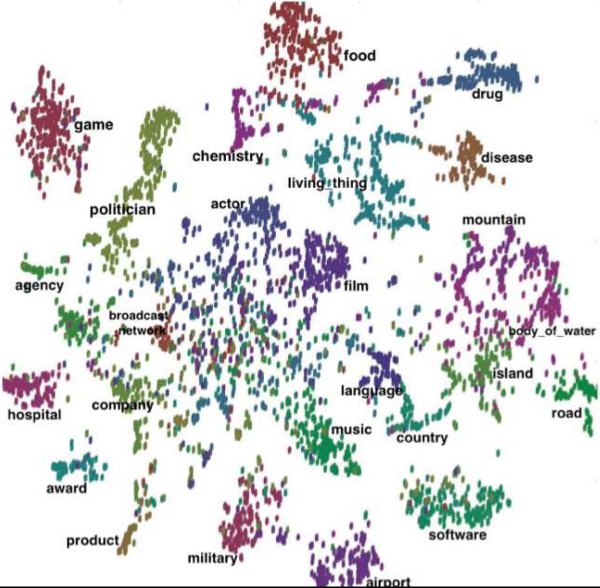


2D projection of embeddings



The semantic field of *supercomputing* in sublexical space.

2D projection of entity embeddings



Outline

word2vec

FastText

- FastText is an extension of word2vec SGNS.
- It also computes embeddings for character ngrams.
- A word's embedding is
  a weighted sum of its character ngram embeddings.
- Parameters: minimum ngram length: 3, maximum ngram length: 6
- The embedding of "dendrite" will be the sum of the following ngrams: @dendrite@ @de den end ndr dri rit ite te@ @den dend endr ndri drit rite ite@ @dend dendr endri ndrit drite rite@ @dendr dendri endrit ndrite drite@

FastText

- Example 1: embedding for character ngram "dendrit"
  $\rightarrow$ "dentrite" and "dentritic" are similar
- Example 2: embedding for character ngram "tech-"
  $\rightarrow$ "tech-rich" and "tech-heavy" are similar

Three frequently used embedding learners

- word2vec
  `https://code.google.com/archive/p/word2vec/`
- FastText
  `https://research.fb.com/projects/fasttext/`
- gensim
  `https://radimrehurek.com/gensim/`

```
fasttext skipgram -dim 50 -input tinycorpus.txt
-output tiny

cat ftvoc.txt | fasttext print-vectors tiny.bin >
ftvoc.vec
```

Letter n-gram generalization can be good

word2vec

1.000 automobile 779 mid-size 770 armored 763 seaplane 754 bus 754 jet 751 submarine 750 aerial 744 improvised 741 anti-aircraft

FastText

1.000 automobile 976 automobiles 929 Automobile 858 manufacturing 853 motorcycles 849 Manufacturing 848 motorcycle 841 automotive 814 manufacturer 811 manufacture

Letter n-gram generalization can be bad

word2vec

1.000 Steelers 884 Expos 865 Cubs 848 Broncos 831 Dinneen 831 Dolphins 827 Pirates 826 Copley 818 Dodgers 814 Raiders

FastText

1.000 Steelers 893 49ers 883 Steele 876 Rodgers 857 Colts 852 Oilers 851 Dodgers 849 Chalmers 849 Raiders 844 Coach

Letter n-gram generalization: no-brainer for unknowns

word2vec

("video-conferences" did not occur in corpus)

FastText

1.000 video-conferences 942 conferences 872 conference 870 Conferences 823 inferences 806 Questions 805 sponsorship 800 References 797 participates 796 affiliations

FastText skipgram parameters

- -input <path>
  training file path
- -output <path>
  output file path
- -lr <float>
  learning rate
- -lrUpdateRate <int>
  rate of updates for the learning rate
- -dim <int>
  dimensionality of word embeddings
- -ws <int>
  size of the context window
- -epoch <int>
  number of epochs

FastText skipgram parameters

- -minCount <int>
  minimal number of word occurences
- -neg <int>
  number of negatives sampled
- -wordNgrams <int>
  max length of word ngram
- -loss <string>
  loss function $\in$ { ns, hs, softmax }
- -bucket <int>
  number of buckets
- -minn <int>
  min length of char ngram
- -maxn <int>
  max length of char ngram

FastText skipgram parameters

- -threads <int>
  number of threads
- -t <float>
  sampling threshold
- -label <string>
  labels prefix
- -verbose <int>
  verbosity level

Takeaway: Three versions of word2vec skipgram

- Matrix factorization (SVD) of PPMI matrix
- skipgram negative sampling (SGNS) using GD
- hierarchical softmax

Takeaway: Embeddings learned via gradient descent

- Cost (actually reward) function is negative sampling:
  Make dot product of "true" pairs as big as possible and of "false" pairs as small as possible
- Number of parameters: $2d|V|$
- Gradient ascent

Takeaway: Visualization

- 2D or 3D visualization of embeddings
- 2D/3D visualization of high-dimensional spaces is often misleading.

Takeaway: FastText

- Learns embeddings for character ngrams
- Can handle out-of-vocabulary (OOV) words
- Sometimes you gain ("automobile"),
  sometimes you lose ("Steelers").