

Feedforward Neural Networks and Word Embeddings

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Outline

- 1 Linear models
- 2 Limitations of linear models
- 3 Neural networks
- 4 A neural language model
- 5 Word embeddings

LINEAR MODELS

Binary Classification with Linear Models

Example: the seminar at < stime > 4 pm will

Classification task: Do we have an < stime > tag in the current position?

Word	Lemma	LexCat	Case	SemCat	Tag
the	the	Art	low		
seminar	seminar	Noun	low		
at	at	Prep	low		stime
4	4	Digit	low		
pm	pm	Other	low	timeid	
will	will	Verb	low		

Feature Vector

Encode context into **feature vector**:

1	bias term		1
2	-3_lemma_the		1
3	-3_lemma_giraffe		0
...	
102	-2_lemma_seminar		1
103	-2_lemma_giraffe		0
...	
202	-1_lemma_at		1
203	-1_lemma_giraffe		0
...	
302	+1_lemma_4		1
303	+1_lemma_giraffe		0
...	

Dot product with weight vector

$$h(X) = X \cdot \Theta^T$$

$$X = \begin{bmatrix} x_0 = 1 \\ x_1 = 1 \\ x_2 = 0 \\ \dots \\ x_{101} = 1 \\ x_{102} = 0 \\ \dots \\ x_{201} = 1 \\ x_{202} = 0 \\ \dots \\ x_{301} = 1 \\ x_{302} = 0 \\ \dots \end{bmatrix}$$

$$\Theta = \begin{bmatrix} w_0 = 1.00 \\ w_1 = 0.01 \\ w_2 = 0.01 \\ \dots \\ x_{101} = 0.01 \\ x_{102} = 0.01 \\ \dots \\ x_{201} = 0.01 \\ x_{202} = 0.01 \\ \dots \\ x_{301} = 0.01 \\ x_{302} = 0.01 \\ \dots \end{bmatrix}$$

Prediction with dot product

$$\begin{aligned}h(X) &= X \cdot \Theta^T \\&= x_0 w_0 + x_1 w_1 + \dots + x_n w_n \\&= 1 * 1 + 1 * 0.01 + 0 * 0.01 + \dots + 0 * 0.01 + 1 * 0.01\end{aligned}$$

Predictions with linear models

Example: the seminar at < stime > 4 pm will

Classification task: Do we have an < stime > tag in the current position?

Linear Model: $h(X) = X \cdot \Theta^T$

Prediction: If $h(X) > 0$, yes. Otherwise, no.

Getting the right weights

Training: Find weight vector Θ such that $h(X)$ is the **correct answer** as many times as possible.

- Given a set T of training examples t_1, \dots, t_n with **correct labels** y_i , find Θ such that $h(X(t_i)) = y_i$ for as many t_i as possible.
- $X(t_i)$ is the feature vector for the i -th training example t_i

Dot product with **trained** weight vector

$$h(X) = X \cdot \Theta^T$$

$$X = \begin{bmatrix} x_0 = 1 \\ x_1 = 1 \\ x_2 = 0 \\ \dots \\ x_{101} = 1 \\ x_{102} = 0 \\ \dots \\ x_{201} = 1 \\ x_{202} = 0 \\ \dots \\ x_{301} = 1 \\ x_{302} = 0 \\ \dots \end{bmatrix}$$

$$\Theta = \begin{bmatrix} w_0 = 1.00 \\ w_1 = 0.001 \\ w_2 = 0.02 \\ \dots \\ w_{101} = 0.012 \\ w_{102} = 0.0015 \\ \dots \\ w_{201} = 0.4 \\ w_{202} = 0.005 \\ \dots \\ w_{301} = 0.1 \\ w_{302} = 0.04 \\ \dots \end{bmatrix}$$

Working with real-valued features

E.g. measure semantic similarity:

Word	sim(time)
the	0.0014
seminar	0.0014
at	0.1
4	2.01
pm	3.02
will	0.5

Working with real-valued features

$$h(X) = X \cdot \Theta^T$$

$$X = \begin{bmatrix} x_0 = 1.0 \\ x_1 = 50.5 \\ x_2 = 52.2 \\ \dots \\ x_{101} = 45.6 \\ x_{102} = 60.9 \\ \dots \\ x_{201} = 40.4 \\ x_{202} = 51.9 \\ \dots \\ x_{301} = 40.5 \\ x_{302} = 35.8 \\ \dots \end{bmatrix}$$

$$\Theta = \begin{bmatrix} w_0 = 1.00 \\ w_1 = 0.001 \\ w_2 = 0.02 \\ \dots \\ x_{101} = 0.012 \\ x_{102} = 0.0015 \\ \dots \\ x_{201} = 0.4 \\ x_{202} = 0.005 \\ \dots \\ x_{301} = 0.1 \\ x_{302} = 0.04 \\ \dots \end{bmatrix}$$

Working with real-valued features

$$\begin{aligned}h(X) &= X \cdot \Theta^T \\&= x_0 w_0 + x_1 w_1 + \dots + x_n w_n \\&= 1.0 * 1 + 50.5 * 0.001 + \dots + 40.5 * 0.1 + 35.8 * 0.04 \\&= 540.5\end{aligned}$$

Working with real-valued features

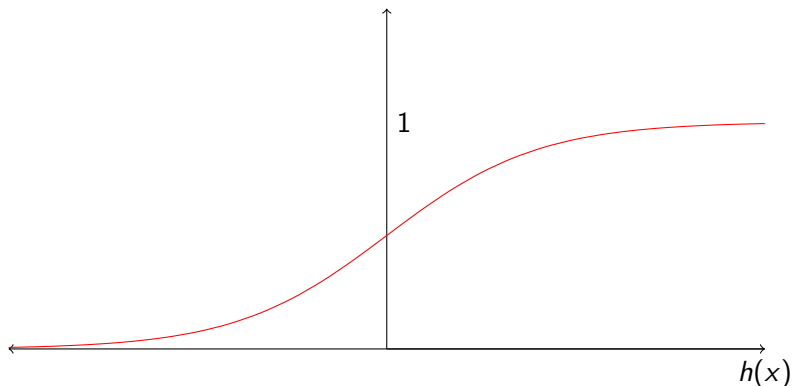
Classification task: Do we have an `< stime >` tag in the current position?

Prediction: $h(X) = 540.5$

- Can we transform this into a probability?

Sigmoid function

We can push $h(X)$ between 0 and 1 using a **non-linear activation** function
The **sigmoid function** $\sigma(Z)$ is often used



Logistic Regression

Classification task: Do we have an `< stime >` tag in the current position?

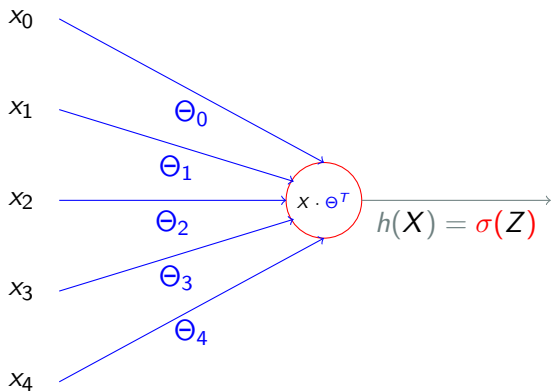
Linear Model: $Z = X \cdot \Theta^T$

Prediction: If $\sigma(Z) > 0.5$, yes. Otherwise, no.

Logistic regression:

- Use a **linear model** and squash values between 0 and 1.
 - ▶ Convert real values to probabilities
- Put threshold to 0.5.
- Positive class above threshold, negative class below.

Logistic Regression

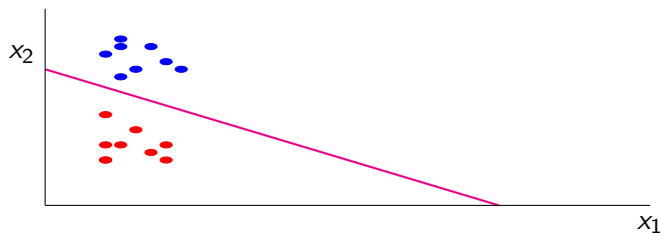


LINEAR MODELS: LIMITATIONS

Decision Boundary

What do **linear** models do?

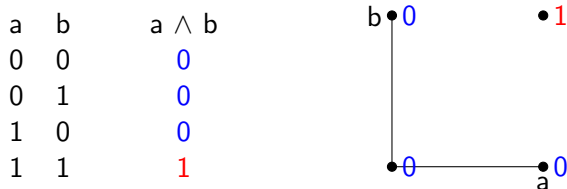
- $\sigma(Z) > 0.5$ when $Z(= X \cdot \Theta^T) \geq 0$
- Model defines a **decision boundary** given by $X \cdot \Theta^T = 0$
 - positive examples (have stime tag)
 - negative examples (no stime tag)



Exercise

When we model a task with linear models, what assumption do we make about positive/negative examples?

Modeling 1: Learning a predictor for \wedge

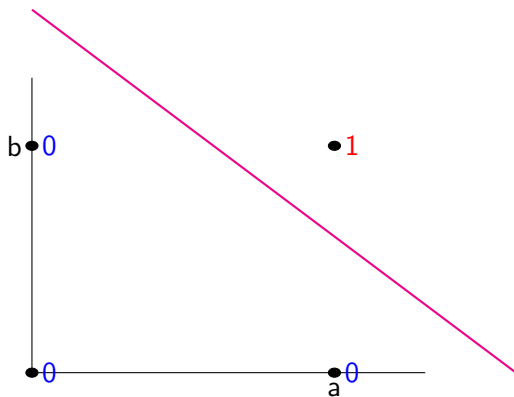


Features : a, b

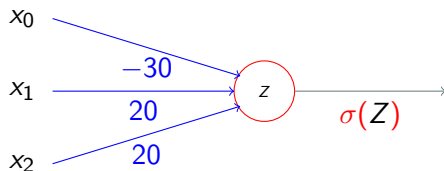
Feature values : binary

Can we learn a linear model to solve this problem?

Modeling 1: Learning a predictor for \wedge



Modeling 1: Logistic Regression



x_0	x_1	x_2	$x_1 \wedge x_2$
1	0	0	$\sigma(1 * -30 + 0 * 20 + 0 * 20) = \sigma(-30) \approx 0$
1	0	1	$\sigma(1 * -30 + 0 * 20 + 1 * 20) = \sigma(-10) \approx 0$
1	1	0	$\sigma(1 * -30 + 1 * 20 + 1 * 20) = \sigma(-10) \approx 0$
1	1	1	$\sigma(1 * -30 + 1 * 20 + 1 * 20) = \sigma(10) \approx 1$

Modeling 2: Learning a predictor for $XNOR$

a	b	a $XNOR$ b
0	0	1
0	1	0
1	0	0
1	1	1

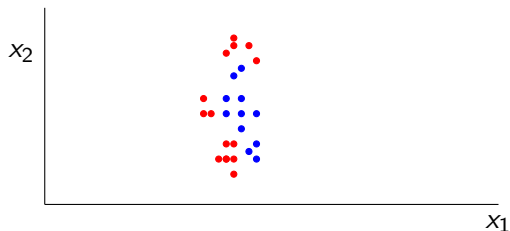


Features : a, b

Feature values : binary

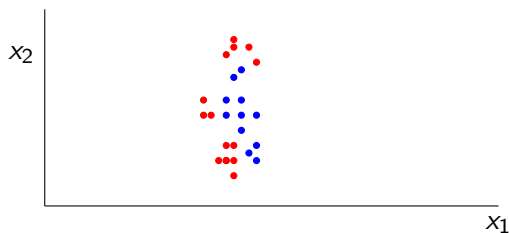
Can we learn a linear model to solve this problem?

Non-linear decision boundaries



Can we learn a linear model to solve this problem?

Non-linear decision boundaries



Can we learn a linear model to solve this problem?

No! Decision boundary is **non-linear**.

Learning a predictor for *XNOR*

Linear models not suited to learn non-linear decision boundaries.

Neural networks can do that.

NEURAL NETWORKS

Learning a predictor for $XNOR$

a	b	a $XNOR$ b
0	0	1
0	1	0
1	0	0
1	1	1

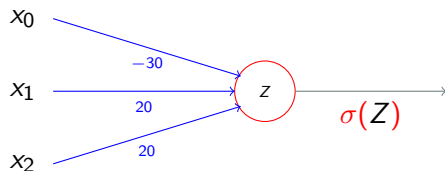
Features : a, b

Feature values : binary

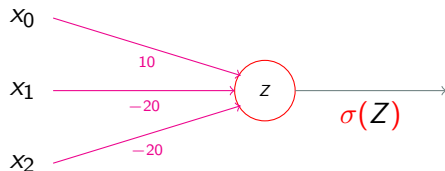
Can we learn a **non-linear model** to solve this problem?

Yes! E.g. through **function composition**.

Function Composition

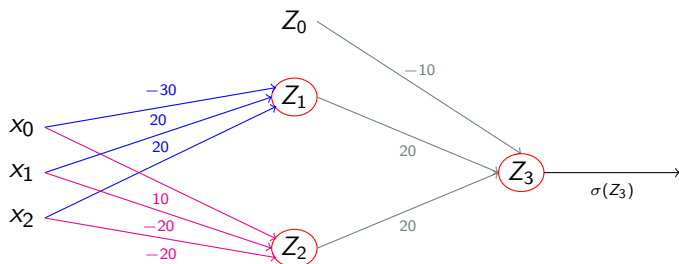


x_0	x_1	x_2	$x_1 \wedge x_2$
1	0	0	≈ 0
1	0	1	≈ 0
1	1	0	≈ 0
1	1	1	≈ 1



x_0	x_1	x_2	$\neg x_1 \wedge \neg x_2$
1	0	0	≈ 1
1	0	1	≈ 0
1	1	0	≈ 0
1	1	1	≈ 0

Function Composition



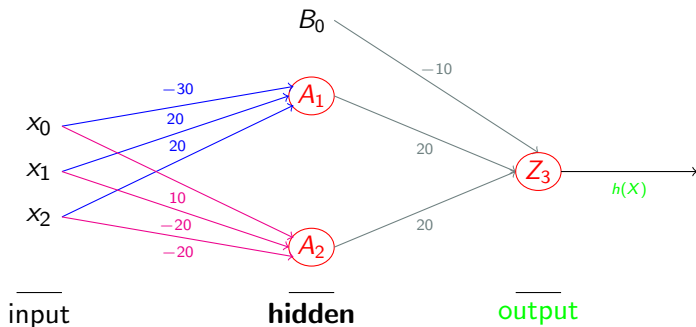
x_0	x_1	x_2	$\sigma(Z_1)$	$\sigma(Z_2)$	$\sigma(Z_3)$
1	0	0	≈ 0	≈ 1	$\sigma(1 * -10 + 0 * 20 + 1 * 20) = \sigma(10) \approx 1$
1	0	1	≈ 0	≈ 0	$\sigma(1 * -10 + 0 * 20 + 0 * 20) = \sigma(-10) \approx 0$
1	1	0	≈ 0	≈ 0	$\sigma(1 * -10 + 0 * 20 + 0 * 20) = \sigma(-10) \approx 0$
1	1	1	≈ 1	≈ 0	$\sigma(1 * -10 + 1 * 20 + 1 * 20) = \sigma(30) \approx 1$

Feedforward Neural Network

We just created a **feedforward neural network** with:

- 1 input layer X (feature vector)
- 2 weight matrices $U = (\Theta_1, \Theta_2)$ and $V = \Theta_3$
- 1 hidden layer H composed of:
 - ▶ 2 activations $A_1 = \sigma(Z_1)$ and $A_2 = \sigma(Z_2)$ where:
 - ★ $Z_1 = X \cdot \Theta_1$
 - ★ $Z_2 = X \cdot \Theta_2$
- 1 output unit $h(X) = \sigma(Z_3)$ where:
 - ▶ $Z_3 = H \cdot \Theta_3$

Feedforward Neural Network



Computation of hidden layer \mathbf{H} :

- $A_1 = \sigma(X \cdot \Theta_1)$
- $A_2 = \sigma(X \cdot \Theta_2)$
- $B_0 = 1$ (bias term)

Computation of output unit $h(X)$:

- $h(X) = \sigma(\mathbf{H} \cdot \Theta_3)$

Feedforward Neural Network

Classification task: Do we have an `< stime >` tag in the current position?

Neural network: $h(X) = \sigma(\mathbf{H} \cdot \Theta_n)$, with:

$$\mathbf{H} = \begin{bmatrix} B_0 = 1 \\ A_1 = \sigma(X \cdot \Theta_1) \\ A_2 = \sigma(X \cdot \Theta_2) \\ \dots \\ A_j = \sigma(X \cdot \Theta_j) \end{bmatrix}$$

Prediction: If $h(X) > 0.5$, yes. Otherwise, no.

Getting the right weights

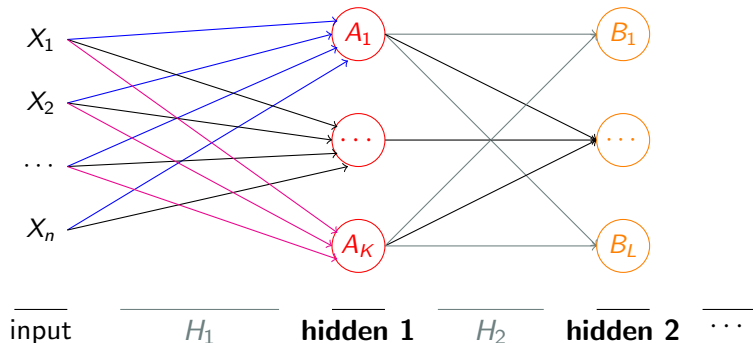
Training: Find weight matrices $U = (\Theta_1, \Theta_2)$ and $V = \Theta_3$ such that $h(X)$ is the **correct answer** as many times as possible.

- Given a set T of training examples t_1, \dots, t_n with **correct labels** \mathbf{y}_i , find $U = (\Theta_1, \Theta_2)$ and $V = \Theta_3$ such that $h(X) = \mathbf{y}_i$ for as many t_i as possible.
 - Computation of $h(X)$ called **forward propagation**
 - Modify $U = (\Theta_1, \Theta_2)$ and $V = \Theta_3$ with error **back propagation**

The intuition behind back propagation is the same as the perceptron update!

Network architectures

Depending on task, a particular network architecture can be chosen:



Note: Bias terms omitted for simplicity

Multi-class classification

- More than two labels
- Instead of “yes” and “no”, predict $c_i \in C = \{c_1, \dots, c_k\}$, where k is the number of classes
- For instance, if we want to detect border tags for `stime` and `etime`, then we don't only have the `<stime>` label but also: `</stime>`, `<etime>`, `</etime>`, **no tag**
- **Use 5 output units** (5 is the number of classes)
 - ▶ Output layer instead of a single output unit
 - ▶ The class with the highest activation is chosen
 - ▶ Probabilities can be obtained by dividing the exponentiated activation for a class by the sum of the exponentiated activations (“softmax”)

Summary: Neural Networks

- We showed how to use neural networks to solve non-linear decision problems
- Neural networks are very powerful - much more powerful than linear models, even more powerful than decision trees
- But we have been working with very simple features (binary features so far in our example).
- Neural networks can combine these simple features into very complex features (as was done previously with feature selection)
- But now we will show how neural language modeling led to the development of very powerful features, “word embeddings”, which are associated with word types

A NEURAL LANGUAGE MODEL

Neural language model

- Early application of neural networks (Bengio et al. 2003)
- Task: Given k previous words, predict the **current word**
Estimate: $P(w_t | w_{t-k}, \dots, w_{t-2}, w_{t-1})$

- Previous (non-neural) approaches:

Problem: Joint distribution of consecutive words difficult to obtain
→ chose small history to reduce complexity ($n=3$)
→ predict for unseen history through back-off to smaller history

Drawbacks:

Takes into account small context

Does not model similarity between words

Word similarity for language modeling

- 1 The cat is walking in the bedroom
 - 2 The dog was running in a room
 - 3 A cat was running in a room
 - 4 A dog was walking in a bedroom
- Model similarity between (cat,dog), (room, bedroom)
- Generalize from 1 to 2 etc.

Neural Language Model (LM)

- **Solution:**

Use **word embeddings** to represent each word in history

→ Each word is represented in relation to the others

→ Distributed word feature vector

Feed to a neural network to learn parameters for the LM task

Feedforward Neural Network for LM

Training example: *The cat is walking in the **bedroom***

Neural network input:

Look at words preceding **bedroom**

→ The cat **is**₋₄ **walking**₋₃ **in**₋₂ **the**₋₁ **bedroom**

→ Create **word embedding** (LT_i) for window

Give LT_i as input to Feedforward Neural Network

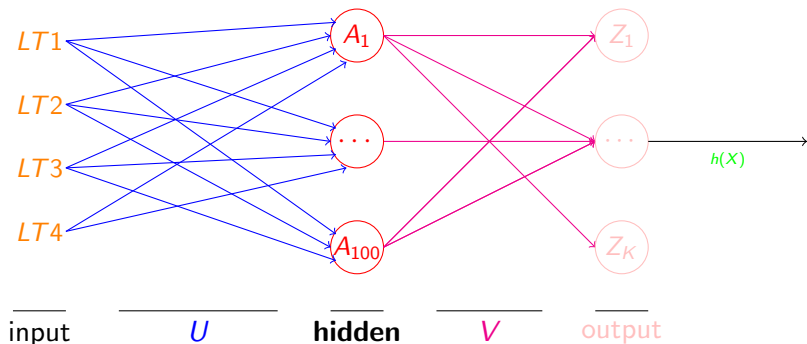
Neural network training:

Predict current word (forward propagation)

→ should be **bedroom**

Train weights by backpropagating error

Feedforward Neural Network for LM



Input: word embeddings LT_i

Output: predicted label (current word)

Note: Bias terms omitted for simplicity

Feedforward Neural Network

Input layer (X): Word features $LT1, LT2, LT3, LT4$

Weight matrices U, V

Hidden layer (H): $\sigma(X \cdot U + d)$

Output layer (O): $H \cdot V + b$

Prediction: $h(X) = \text{softmax}(O)$

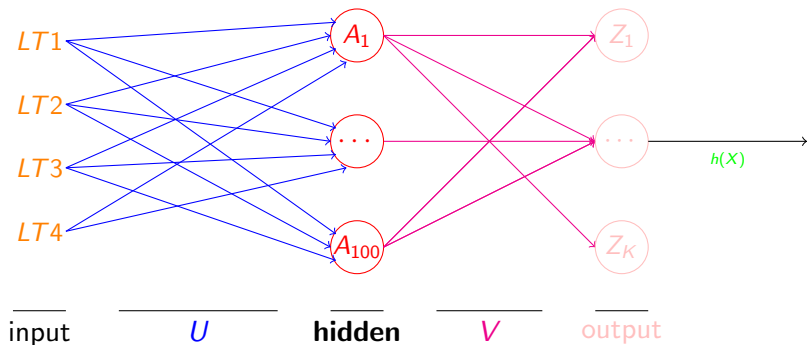
- Predicted class is the one with highest probability (given by softmax)

Weight training

Training: Find weight matrices U and V such that $h(X)$ is the **correct answer** as many times as possible.

- Given a set T of training examples t_1, \dots, t_n with **correct labels** y_i , find U and V such that $h(X) = y_i$ for as many t_i as possible.
- Computation of $h(X)$ with **forward propagation**
- U and V with error **back propagation**

Forward Propagation



Forward propagation:

→ Perform all operations to get $h(X)$ from input LT .

Forward Propagation

Input layer (X): Word features $LT1, LT2, LT3, LT4$

Weight matrices U, V

Hidden layer (H): $\sigma(X \cdot U + d)$

Output layer (O): $H \cdot V + b$

Prediction: $h(X) = \text{softmax}(O)$

- Predicted class is the one with highest probability (given by softmax)

Backpropagation

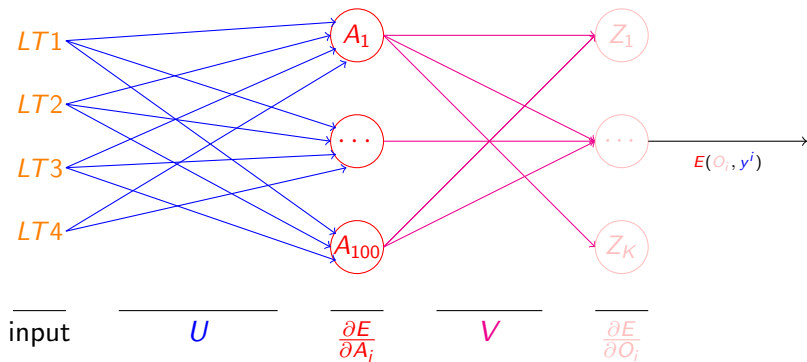
Goal of training: adjust weights such that **correct label is predicted**

→ **Error** between **correct label** and **prediction** is minimal

Sketch:

- Convert **difference** between **prediction** and **error** into **derivatives**
- Compute **derivatives** in each **hidden layer** from **layer above**
 - ▶ **Backpropagate** the error derivative with respect to the output of a unit
- Use derivatives with respect to **the activations** to get error derivatives with respect to **incoming weights**

Backpropagation



Backpropagation:

→ Compute E

→ Compute $\frac{\partial E}{\partial O_i}$

Backpropagation

Compute **error at output E**:

Compare **output unit** with y^i

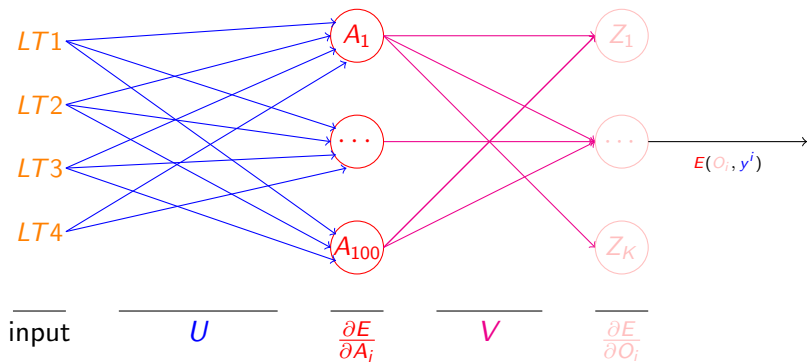
▶ y^i vector with 1 in correct class, 0 otherwise

$$E = \frac{1}{2} \sum_{i=1}^n (y_i - O_i)^2 \text{ (mean squared)}$$

Compute $\frac{\partial E}{\partial O_i}$:

$$\frac{\partial E}{\partial O_i} = -(y_i - O_i)$$

Backpropagation



Backpropagation:

→ Compute $\frac{\partial E}{\partial A_j}$

Backpropagation

Compute **derivatives** in each hidden layer from layer above:

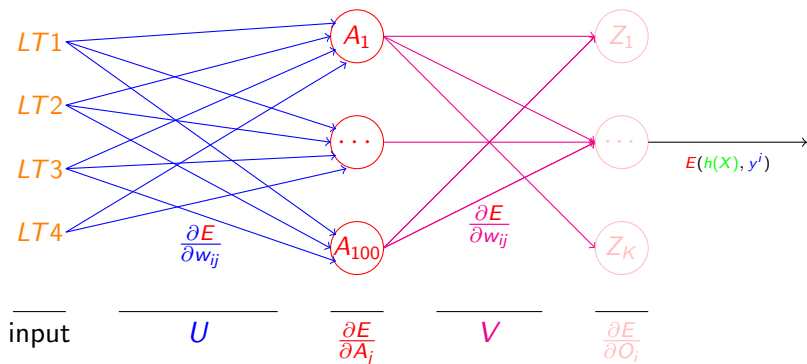
Compute derivative of **error** with respect to **logit** (output)

Compute derivative of **error** with respect to **previous hidden unit**

Compute derivative with respect to **weights**

→ Use **recursion** to do this for every layer

Backpropagation



Weight training

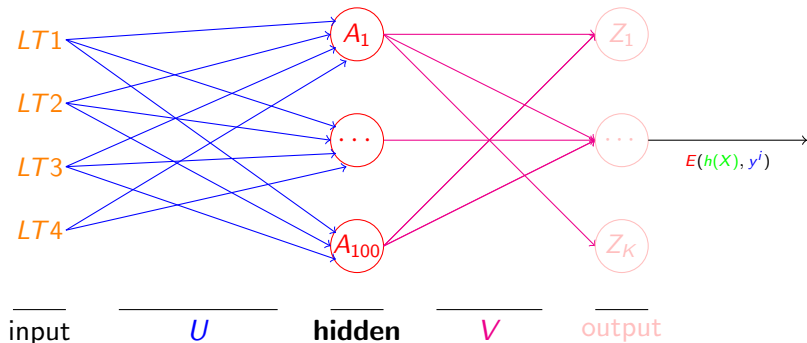
Training: Find weight matrices U and V such that $h(X)$ is the **correct answer** as many times as possible.

- Computation of $h(X)$ with **forward propagation**
- U and V with error **back propagation**

For each batch of training examples

- 1 Forward propagation to get predictions
- 2 Backpropagation of error
 - ▶ Gives gradient of E given **input**
- 3 Modify weights (gradient descent)
- 4 Goto 1 until convergence

Word Embedding Layer



Word Embedding Layer

- Each word type encoded into index vector $w_i = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$
- LT_i is dot product of **weight matrix C** with index of w_i
→ C is **shared**. Each column in C is used for all words (tokens) of a particular word-type.

Dot product with (trained) weight vector

$W = \{\text{the, cat, on, table, chair}\}$

$$w_{table} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 0.02 & 0.1 & 0.05 & 0.03 & 0.01 \\ 0.15 & 0.2 & 0.01 & 0.02 & 0.11 \\ 0.03 & 0.1 & 0.04 & 0.04 & 0.12 \end{bmatrix}$$

$$LT_{table} = w_{table} \cdot C^T = \begin{bmatrix} 0.03 \\ 0.02 \\ 0.04 \end{bmatrix}$$

Words get mapped to lower dimension

→ Hyperparameter to be set

Dot product with (initial) weight vector

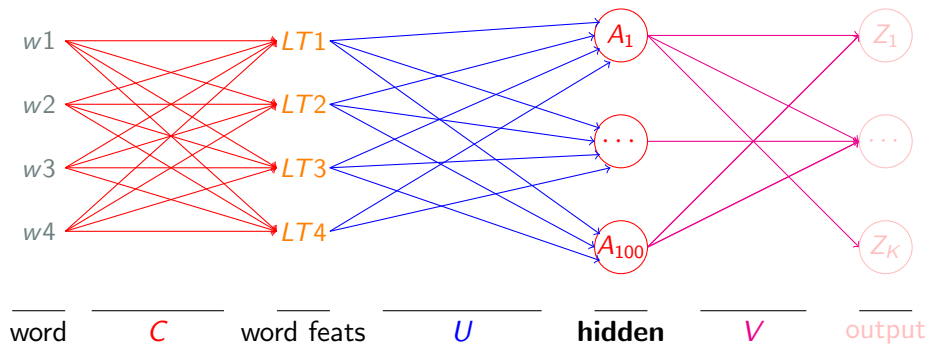
$W = \{\text{the, cat, on, table, chair}\}$

$$w_{table} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 0.01 & 0.01 & 0.01 & 0.01 & 0.01 \\ 0.01 & 0.01 & 0.01 & 0.01 & 0.01 \\ 0.01 & 0.01 & 0.01 & 0.01 & 0.01 \end{bmatrix}$$

$$LT_{table} = w_{table} \cdot C^T = \begin{bmatrix} 0.01 \\ 0.01 \\ 0.01 \end{bmatrix}$$

Feature vectors same for all words.

Feedforward Neural Network with Lookup Table



Note: Bias terms omitted for simplicity

Weight training

Training: Find weight matrices C , U and V such that $h(X)$ is the **correct answer** as many times as possible.

- Given a set T of training examples t_1, \dots, t_n with **correct labels** \mathbf{y}_i , find C , U and V such that $h(X) = \mathbf{y}_i$ for as many t_i as possible.
- Computation of $h(X)$ with **forward propagation**
- Modify C , U and V with error **back propagation**

Dot product with (trained) weight matrix

$W = \{\text{the, cat, on, table, chair}\}$

$$W_{\text{table}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 0.02 & 0.1 & 0.05 & 0.03 & 0.01 \\ 0.15 & 0.2 & 0.01 & 0.02 & 0.11 \\ 0.03 & 0.1 & 0.04 & 0.04 & 0.12 \end{bmatrix}$$

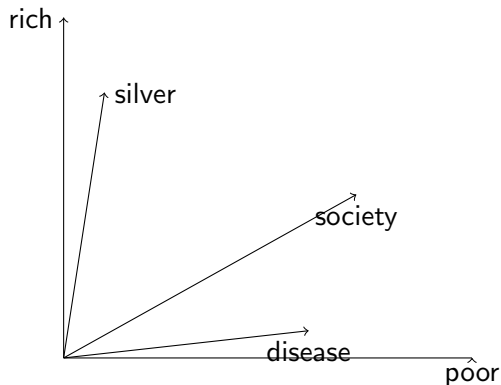
$$LT_{\text{table}} = W_{\text{table}} \cdot C^T = \begin{bmatrix} 0.03 \\ 0.02 \\ 0.04 \end{bmatrix}$$

Each word type gets a **specific** feature vector

WORD EMBEDDINGS

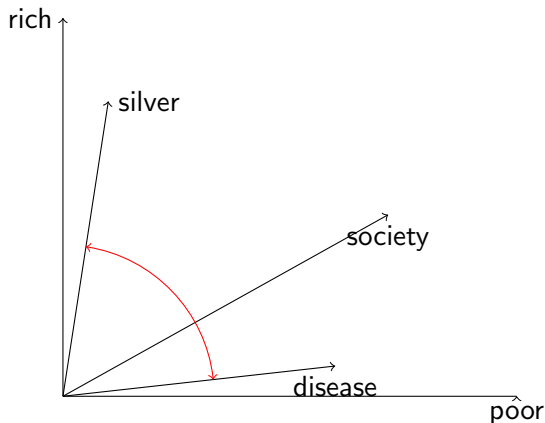
Word Embeddings

- Representation of words in vector space



Word Embeddings

- Similar words are close to each other
→ Similarity is the cosine of the angle between two word vectors



Underlying thoughts

- Assume the equivalence of:
 - ▶ Two words are **semantically similar**.
 - ▶ Two words occur in **similar contexts** (Miller & Charles, roughly).
 - ▶ Two words have **similar word neighbors** in the corpus.
- Elements of this are from Leibniz, Harris, Firth, and Miller.
- Strictly speaking, similarity of neighbors is neither necessary nor sufficient for semantic similarity.
- But perhaps this is **good enough**.

Adapted slide from Hinrich Schütze

Learning word embeddings

Count-based methods:

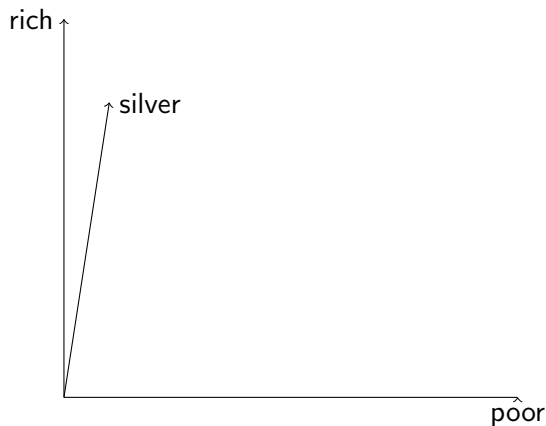
- Compute cooccurrence statistics
- Learn high-dimensional representation
- Map sparse high-dimensional vectors to small dense representation

Word cooccurrence in Wikipedia

- corpus = English Wikipedia
- cooccurrence defined as **occurrence within $k = 10$ words** of each other
 - ▶ $\text{cooc.}(\text{rich}, \text{silver}) = 186$
 - ▶ $\text{cooc.}(\text{poor}, \text{silver}) = 34$
 - ▶ $\text{cooc.}(\text{rich}, \text{disease}) = 17$
 - ▶ $\text{cooc.}(\text{poor}, \text{disease}) = 162$
 - ▶ $\text{cooc.}(\text{rich}, \text{society}) = 143$
 - ▶ $\text{cooc.}(\text{poor}, \text{society}) = 228$

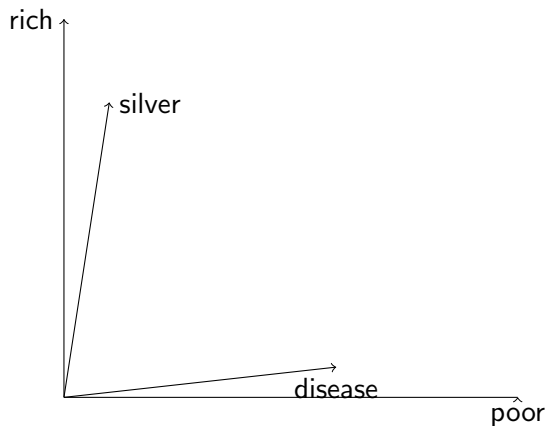
Adapted slide from Hinrich Schütze

Cooccurrence-based Word Space



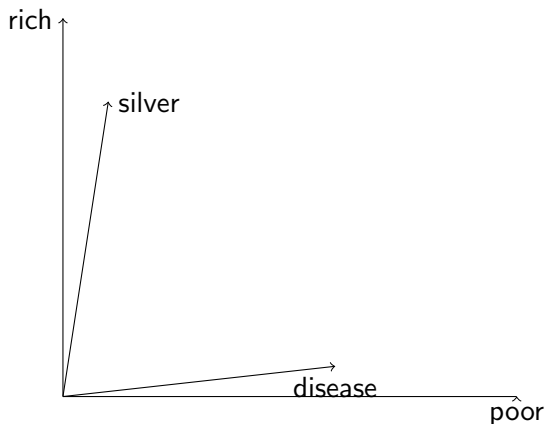
$$\text{cooc.}(\text{poor}, \text{silver}) = 34, \text{cooc.}(\text{rich}, \text{silver}) = 186$$

Cooccurrence-based Word Space



$\text{cooc.}(\text{poor}, \text{disease}) = 162, \text{cooc.}(\text{rich}, \text{disease}) = 17.$

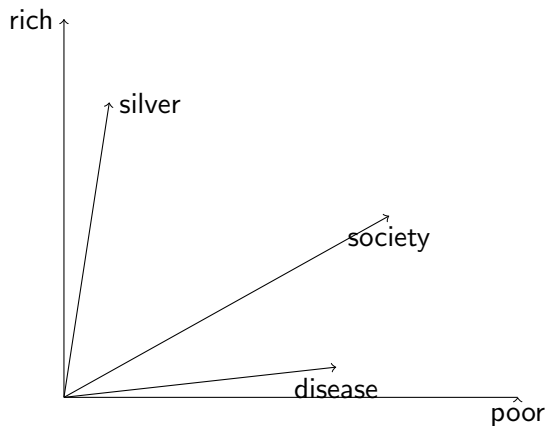
Exercise



$\text{cooc.}(\text{poor}, \text{society})=228$, $\text{cooc.}(\text{rich}, \text{society})=143$

How is it represented?

Cooccurrence-based Word Space



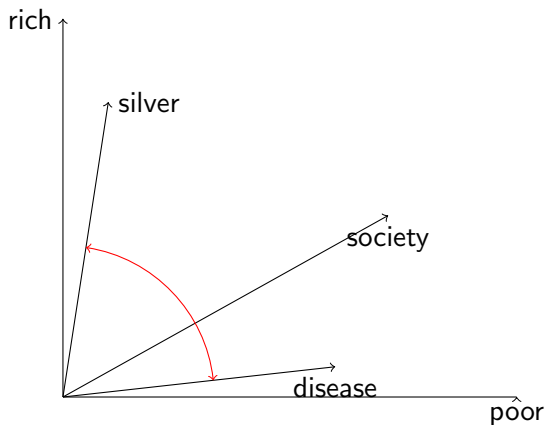
$\text{cooc.}(\text{poor}, \text{society})=228, \text{cooc.}(\text{rich}, \text{society})=143$

Dimensionality of word space

- Up to now we've only used two dimension words: rich and poor.
- Do this for all possible words in a corpus → **high-dimensional space**
- Formally, there is no difference to a two-dimensional space with three vectors.
- Note: a word can have a **dual role** in word space.
 - ▶ Each word can, in principle, be a **dimension word**, an axis of the space.
 - ▶ But each word is also a **vector** in that space.

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Semantic similarity



Similarity is the cosine of the angle between two word vectors

Learning word embeddings

Count-based methods:

- Compute cooccurrence statistics
- Learn high-dimensional representation
- Map sparse high-dimensional vectors to small dense representation

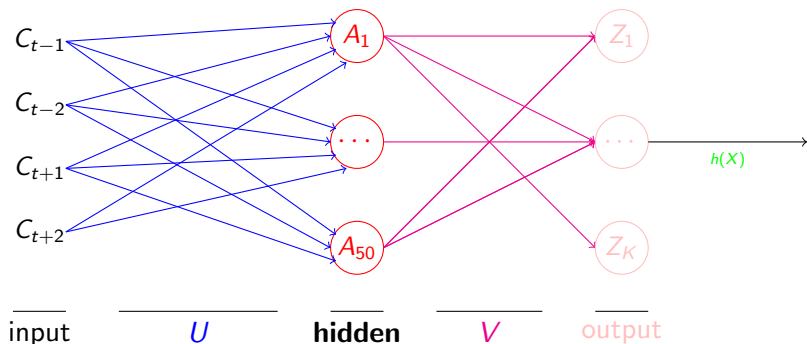
Neural networks:

- Predict a word from its neighbors
- Learn (small) embedding vectors

Word vectors with Neural Networks

- LM Task: Given k previous words, predict the current word
 - For each word w in V , model $P(w_t | w_{t-1}, w_{t-2}, \dots, w_{t-n})$
 - **Learn embeddings C of words**
- Word embeddings learning task: Given k context words, predict the current word
 - **Learn embeddings C of words**

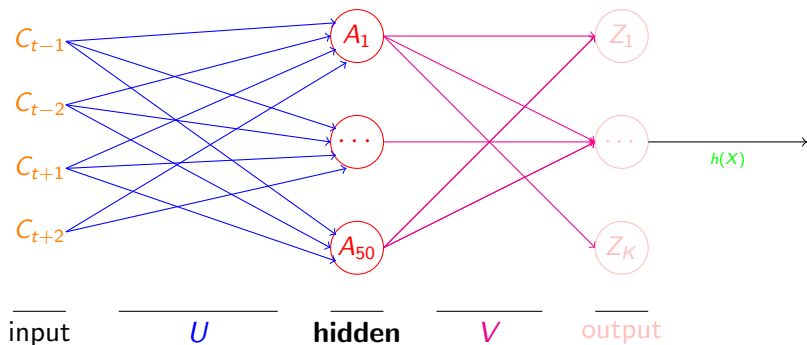
Network architecture



Given words w_{t-2} , w_{t-1} , w_{t+1} and w_{t+2} , predict w_t ("CBOW")

Note: Bias terms omitted for simplicity

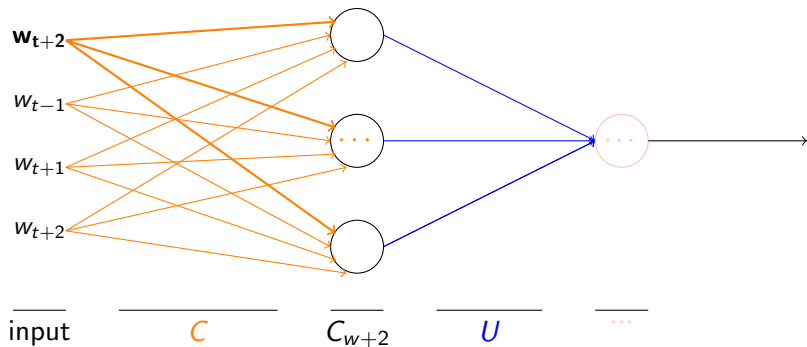
Network architecture



We want the **context vectors** \rightarrow embed words in shared space

Note: Bias terms omitted for simplicity

Getting the Word Embeddings

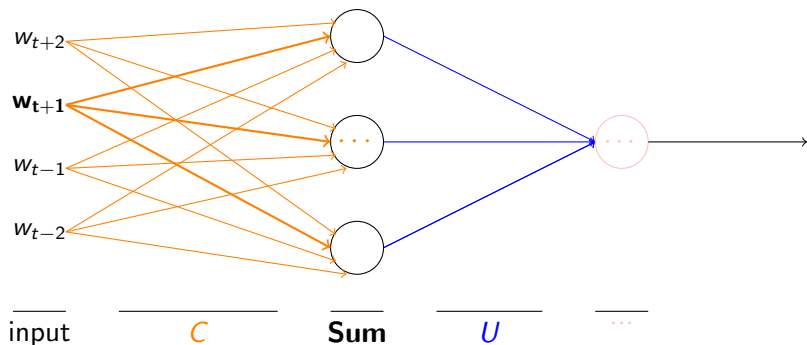


Note: Bias terms omitted for simplicity

Simplifications

- Remove hidden layer
- Sum over all projections

Simplifications



Remove hidden layer and sum over context

Note: Bias terms omitted for simplicity

Simplifications

- Single **logistic unit** instead of output layer
 - No need for distribution over words (only vector representation)
 - Task as binary classification problem:
 - ▶ Given input and weight matrix say if w_t is current word
 - ▶ We know the correct w_t , how do we get the wrong ones?
 - **negative sampling**

- BOW model (Mikolov. 2013)
- Skip-gram model:
 - ▶ Input is w_t
 - ▶ Prediction is w_{t+2} , w_{t+1} , w_{t-1} and w_{t-2}

Applications

Semantic similarity:

- How similar are the words:
 - ▶ *coast* and *shore*; *rich* and *money*; *happiness* and *disease*; *close* and *open*
- WordSim-353 (Finkelstein et al. 2002)
 - ▶ Measure associations
- SimLex-999
 - ▶ Only measure semantic similarity

Other tasks:

- Use word embeddings as input features for other tasks (e.g. sentiment analysis, language modeling, named entity recognition)

Recap

- Cannot fit data with **non-linear** decision boundary with linear models

Solution: compose non-linear functions with **neural networks**

→ Successful in many NLP applications:

- ▶ Language modeling
 - ▶ Learning word embeddings
- Feeding word embeddings into neural networks has proven successful in many NLP tasks, e.g.:
 - ▶ Sentiment Analysis
 - ▶ Named Entity Recognition

Thank you !

Backpropagation - Details

- The next slide shows the actual computation of backpropagation, showing the derivatives that are computed.
- The actual updates are also shown, these are more intuitive than the derivatives for many people.

Backpropagation

Compute **derivatives** in each hidden layer from **layer above**:

Compute derivative of **error** with respect to **logit** (output)

$$\frac{\partial E}{\partial Z_i} = \frac{\partial E}{\partial O_i} \frac{\partial O_i}{\partial Z_i} = \frac{\partial E}{\partial O_i} O_i(1 - O_i) \quad (\text{Note: } O_i = \frac{1}{1+e^{-Z_i}})$$

Compute derivative of **error** with respect to **previous hidden unit**

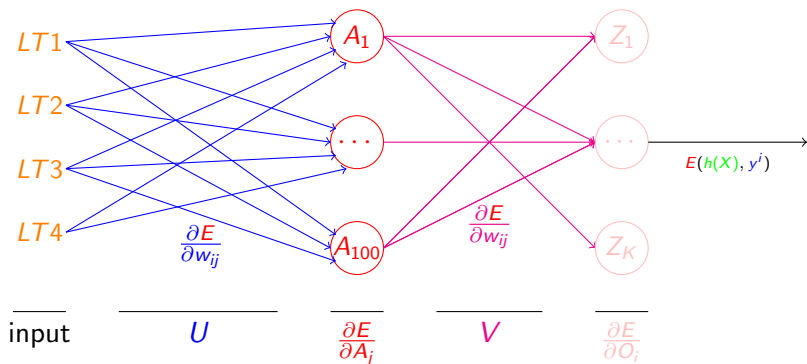
$$\frac{\partial E}{\partial A_j} = \sum_i \frac{\partial Z_i}{\partial A_j} \frac{\partial E}{\partial Z_i} = \sum_i w_{ji} \frac{\partial E}{\partial Z_i}$$

Compute derivative with respect to **weights**

$$\frac{\partial E}{\partial w_{ji}} = \frac{\partial Z_i}{\partial w_{ji}} \frac{\partial E}{\partial Z_i} = O_j \frac{\partial E}{\partial Z_i}$$

→ Use **recursion** to do this **for every layer**

Backpropagation



Weight training

Training: Find weight matrices U and V such that $h(X)$ is the **correct answer** as many times as possible.

- Computation of $h(X)$ with **forward propagation**
- U and V with error **back propagation**

For each batch of training examples

- 1 Forward propagation to get predictions
- 2 Backpropagation of error
 - ▶ Gives gradient of E given **input**
- 3 Modify weights (gradient descent)
- 4 Goto 1 until convergence