

Einführung in die Computerlinguistik

HMMs

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Center for Information and Language Processing

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Outline

- 1 StatNLP
- 2 Basics
- 3 POS tagging
- 4 POS setup
- 5 Probabilistic POS tagging
- 6 Viterbi

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Statistical Natural Language Processing

Definition

Statistical Natural Language Processing (StatNLP) uses methods of supervised, semisupervised and unsupervised learning to address tasks that involve written or spoken (human) language.

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statistics = the practice or science of collecting and analyzing numerical data

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statistics vs. machine learning

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speech recognition

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optical character recognition

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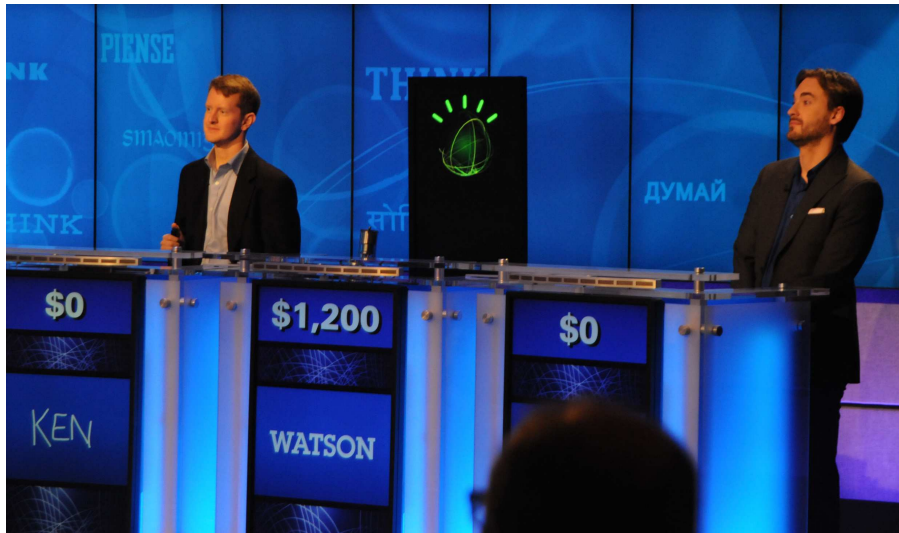
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 - a small group of researchers that do active research on machine learning methods

Recent big success story 1

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Recent big success story 2

Recent big success story 2



Siri. beta Your wish is its command.

Siri on iPhone 4S lets you use your voice to send messages, schedule meetings, place phone calls, and more. Ask Siri to do things just by talking the way you talk. Siri understands what you say, knows what you mean, and even talks back. Siri is so easy to use and does so much, you'll keep finding more and more ways to use it.



Recent big success story 3

Google Translate – more on this later

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max, argmax

max

$\max_x f(x)$

the largest value of $f(x)$

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- $\max_x (-(x-2)^2 + 5)$
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Positive factor $c > 0$ does not affect argmax

$$\operatorname{argmax}_x f(x) = \operatorname{argmax}_x c \cdot f(x)$$

$$\operatorname{argmax}_x f(x) = \operatorname{argmax}_x 1/c \cdot f(x)$$

Σ

$$\sum_{i=m}^{i=n} f(i) = f(m) + f(m+1) + \dots + f(n-1) + f(n)$$

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Joint probability

- The joint probability $P(AB)$ is the probability that A and B occur together / at the same time (i.e., jointly).

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- We can write $P(AB)$ as $P(A \cap B)$ if A and B are formalized as sets.

Conditional probability

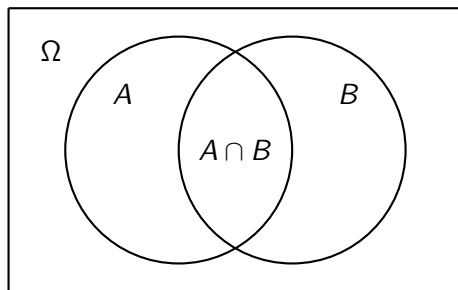
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- Definition: $P(A|B) = \frac{P(AB)}{P(B)}$ ($P(B) > 0$)

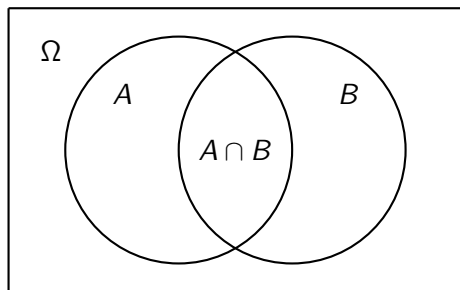
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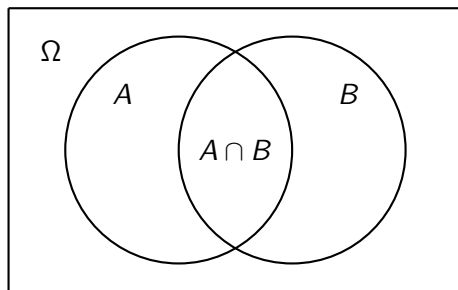
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To compute $P(A|B)$: Divide the area of $A \cap B$ by the area of B .
$$P(A|B) = P(A \cap B) / P(B)$$

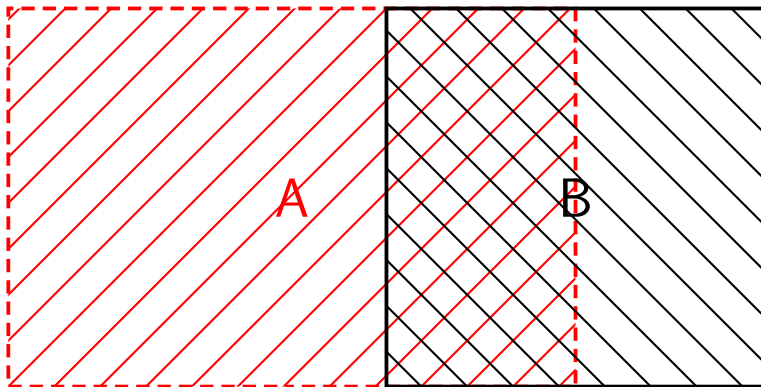
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$$P(B|A) = P(A \cap B) / P(A)$$



Compute $P(A|B) = P(A \cap B)/P(B)$ and
 $P(B|A) = P(A \cap B)/P(A)$

$$P(X_1 X_2 X_3 \dots X_n) =$$

$$P(X_1) \cdot P(X_2|X_1) \cdot P(X_3|X_1 X_2) \cdot \dots \cdot P(X_n|X_1 X_2 \dots X_{n-1})$$

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- Follows from
$$P(A) = P(AB) + P(A\bar{B}) = P(A|B)P(B) + P(A|\bar{B})P(\bar{B})$$

- Two events A and B are independent iff $P(AB) = P(A)P(B)$

Independence

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- If I learn that A is true, then that doesn't change my assessment of the probability of B (and vice versa).
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 $P(A) = P(A|B)$, $P(B) = P(B|A)$

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- Why \approx ?

Testing for independence: Example

A = champagne, B = sparkling

Übung

Find either two independent words or two words that occur less often on the same page than expected by chance

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- In the context “the book” it can only be a noun.
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- Part-of-speech tagging assigns to “book” the correct syntactic category in context.

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- Are all cases of part-of-speech tagging this easy?

Hard example

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In this case, finding the correct parts of speech for the sentence is more difficult.

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- Part-of-speech tagging is used as a **preprocessing step**.
- It is solvable: Very high accuracy rates can be achieved (95–98%).
- It helps with many things you want to do with text, e.g., chunking, information extraction, question answering and parsing.

Part-of-speech tagging of tweets

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ikr	smh	he	asked	fir	yo	last
!	G	O	V	P	D	A
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Tagging is a preprocessing step for many NLP tasks.

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BEZ	the word “is”
IN	preposition
JJ	adjective
JJR	comparative adjective
MD	modal
NN	singular or mass noun
NNP	singular proper noun
NNS	plural noun
PERIOD	. : ? !
PN	personal pronoun

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VBZ	verb, 3rd singular present
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Tag: “Peter arrived in London on Tuesday”

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- What information is available to disambiguate this sentence syntactically?

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 - Example: for a JJ/NN ambiguity in the context “AT _ VBZ”, NN is much more likely than JJ.
- 2 A word's **bias** for the different parts of speech
 - Example: “put” is much more likely to occur as a VBD than as an NN.

Information sources

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- This source of information lets us do 90% correct tagging of English very easily: Just pick the most frequent tag for each word.
- For most words in English, the distribution of tags is very **uneven**: there is one very frequent tag and the others are rare.

Notation

Notation

w_i	the word at position i in the corpus
t_i	the tag of w_i
w^l	the l^{th} word in the lexicon
t^j	the j^{th} tag in the tag set
$C(w^l)$	the number of occurrences of w^l in the training set
$C(t^j)$	the number of occurrences of t^j in the training set
$C(t^j t^k)$	the number of occurrences of t^j followed by t^k
$C(w^l : t^j)$	the number of occurrences of w^l that are tagged as t^j

Notation: Example

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the	representative	put	chairs	on	the	table
w_1	w_2	w_3	w_4	w_5	w_6	w_7
w^5	w^{81}	w^3	w^4	w^1	w^5	w^6
AT	NN	VBD	NNS	IN	AT	NN
article	noun	verb-d	noun-s	prep	article	noun
t_1	t_2	t_3	t_4	t_5	t_6	t_7
t^{16}	t^{12}	t^2	t^9	t^3	t^{16}	t^{12}

$$\begin{array}{lcl}
 C(w^5) & = & 2 \\
 C(t^{16}) & = & 2 \\
 C(t^{16}t^{12}) & = & 2 \\
 C(t^{16}t^2) & = & 0 \\
 C(w^5 : t^{16}) & = & 2
 \end{array}
 \quad
 \begin{array}{lcl}
 C(w^4) & = & 1 \\
 C(t^2) & = & 1 \\
 C(t^{12}t^2) & = & 1 \\
 C(w^5w^{81}) & = & 1 \\
 C(w^5 : t^{12}) & = & 0
 \end{array}$$

Confidence/NN in/IN the/AT pound/NN is/BEZ widely/RB
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for/IN release/NN tomorrow/NN ,/, fail/VB to/TO show/VB
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Chancellor/NNP of/IN the/AT Exchequer/NNP Nigel/NNP
Lawson/NNP 's/POS restated/VBN commitment/NN to/TO
a/AT firm/JJ monetary/JJ policy/NN has/VBZ helped/VBN
to/TO prevent/VB a/AT freefall/NN in/IN sterling/NN over/IN
the/AT past/JJ week/NN ./.

Give the values of the following: w_4 , t_5 , $C(w_8)$, $C(t_9)$, $C(t_1 t_2)$,
 $C(w_3 : t_3)$

Supervised learning

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- Apply statistical model to new text that we want to analyze for some task (information retrieval, machine translation etc)

Tagged training corpus/set: Example

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Outline

- 1 StatNLP
- 2 Basics
- 3 POS tagging
- 4 POS setup
- 5 Probabilistic POS tagging**
- 6 Viterbi

Contents of this section

- Parameter estimation: context parameters
- Parameter estimation: bias parameters
- Greedy tagging
- Viterbi tagging

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- Note that this is a very impoverished model of context.
 - Limited horizon, Markov assumption: we assume that our memory is limited to a [single preceding tag](#).
 - Time invariance, stationary: we assume that these conditional probabilities don't change. (e.g., the same at the beginning and at the end of the sentence)

Parameter estimation: Context

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•

$$\hat{P}_{ml}(\text{NN}|\text{JJ}) = \frac{C(\text{JJ NN})}{C(\text{JJ})}$$

n^{th} order Markov model

In an n^{th} order Markov model,
the tag at time t depends on the n previous tags.

- Order 0: Tag does not depend on previous tags.
- Order 1: Tag depends on immediately preceding tag.
- Order 2: Tag depends on two immediately preceding tags.
- Order 3: Tag depends on three immediately preceding tags.
- ...

(analogous for Markov model that emits words instead of tags)

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Parameter estimation: Word bias

- How to estimate $P(\text{book}|\text{NN})$
-

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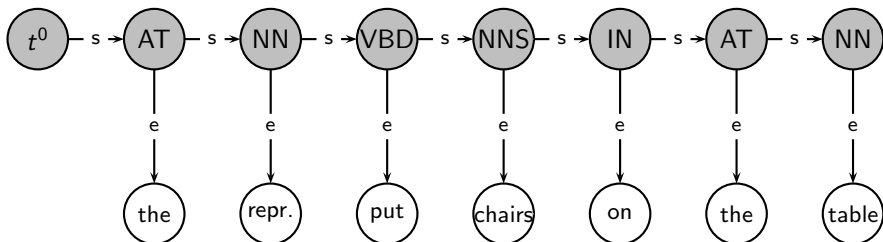
Estimate $P(\text{take}|\text{VB})$ and $P(\text{AT}|\text{IN})$

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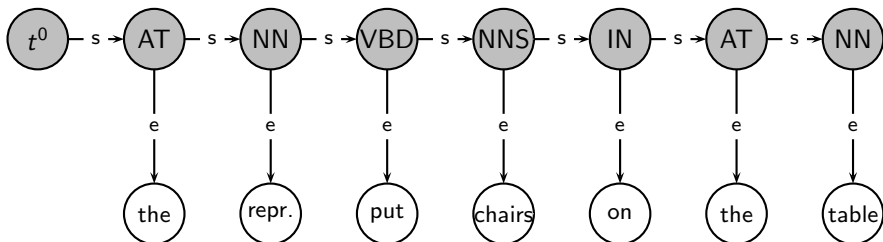
$P(w|t)$ versus $P(t|w)$

(s = sequence, e = emission)



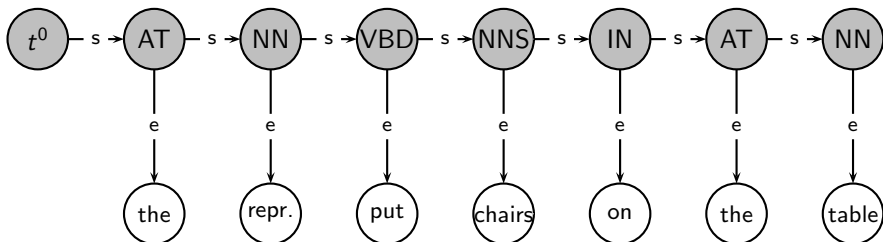
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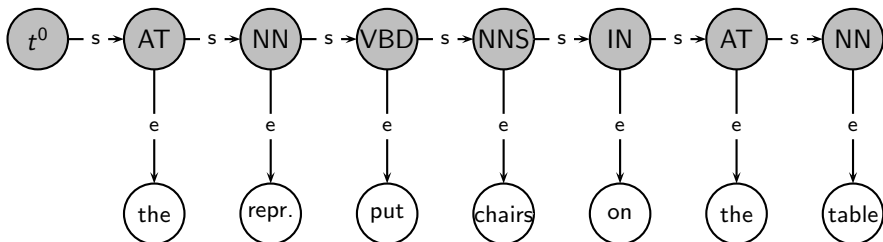
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- $t_3 = \text{VBD}$ maximizes $P(t_3|\text{NN})P(\text{put}|t_3)$

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- What can go wrong with greedy tagging?
- Example?
- A representative put costs 20% more today than a month ago.

Notation (2)

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w_i	the word at position i in the corpus
t_i	the tag of w_i
$w_{i,i+m}$	the words occurring at positions i through $i+m$ (alternative notations: $w_i \cdots w_{i+m}$, w_i, \dots, w_{i+m} , $w_{i(i+m)}$)
$t_{i,i+m}$	the tags $t_i \cdots t_{i+m}$ for $w_i \cdots w_{i+m}$
w^l	the l^{th} word in the lexicon
t^j	the j^{th} tag in the tag set
$C(w^l)$	the number of occurrences of w^l in the training set
$C(t^j)$	the number of occurrences of t^j in the training set
$C(t^j t^k)$	the number of occurrences of t^j followed by t^k
$C(w^l : t^j)$	the number of occurrences of w^l that are tagged as t^j
T	number of tags in tag set
W	number of words in the lexicon
n	sentence length

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- We define our goal thus: Given a sentence, find the most probable sequence of tags for this sentence.
- Formalization of this goal:

$$t_{1,n} = \operatorname{argmax}_{t_{1,n}} P(t_{1,n} | w_{1,n})$$

Simplifying the argmax (1)

$$t_{1,n} = \operatorname{argmax}_{t_{1,n}} P(t_{1,n} | w_{1,n}) \quad (1)$$

$$= \operatorname{argmax}_{t_{1,n}} P(t_{0,n} | w_{1,n}) \quad (2)$$

$$= \operatorname{argmax}_{t_{1,n}} \frac{P(w_{1,n} | t_{0,n}) P(t_{0,n})}{P(w_{1,n})} \quad (3)$$

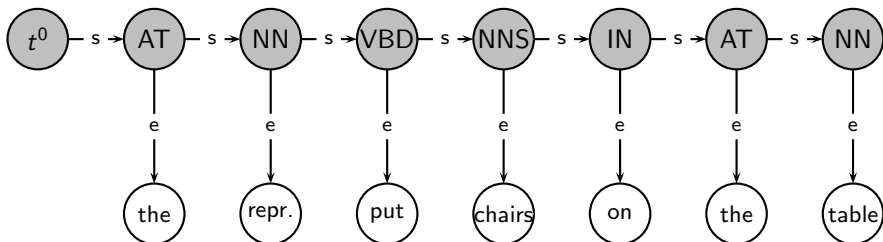
$$= \operatorname{argmax}_{t_{1,n}} P(w_{1,n} | t_{0,n}) P(t_{0,n}) \quad (4)$$

$$= \operatorname{argmax}_{t_{1,n}} \left[\prod_{i=1}^n P(w_i | t_{0,n}) \right] P(t_{0,n}) \quad (5)$$

2: dummy “start” tag; 3: Bayes; 4: positive factor doesn't affect argmax; 5: assumption: words are independent

$P(w|t)$ versus $P(t|w)$

(s = sequence, e = emission)



- The tags generate the words (not vice versa).
- Hence: The tags are given and the words are conditioned on the tags ...
- ... and the correct formalization is $P(w|t)$.

Simplifying the argmax (2)

$$= \operatorname{argmax}_{t_{1,n}} \left[\prod_{i=1}^n P(w_i | t_i) \right] P(t_{0,n}) \quad (6)$$

$$= \operatorname{argmax}_{t_{1,n}} \left[\prod_{i=1}^n P(w_i | t_i) \right] \left[\prod_{i=1}^n P(t_i | t_{0,i-1}) \right] \quad (7)$$

$$= \operatorname{argmax}_{t_{1,n}} \left[\prod_{i=1}^n P(w_i | t_i) \right] \left[\prod_{i=1}^n P(t_i | t_{i-1}) \right] \quad (8)$$

$$= \operatorname{argmax}_{t_{1,n}} \prod_{i=1}^n [P(w_i | t_i) P(t_i | t_{i-1})] \quad (9)$$

7: chain rule; 8: Markov assumption; 9:

$$\prod_{i=1}^n x_i \prod_{i=1}^n y_i = \prod_{i=1}^n x_i y_i$$

Simplifying the argmax (3)

$$= \operatorname{argmax}_{t_{1,n}} \prod_{i=1}^n [P(w_i|t_i)P(t_i|t_{i-1})] \quad (10)$$

$$= \operatorname{argmax}_{t_{1,n}} \sum_{i=1}^n [\log P(w_i|t_i) + \log P(t_i|t_{i-1})] \quad (11)$$

11: computation in log space more efficient / convenient

Simplifying the argmax (3)

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Do you recognize these parameters?

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The most probable tag sequence (= tagging)

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What's the difficulty if you want to tag based on this?

Brute force is very inefficient

$$\operatorname{argmax}_{t_{1,n}} \sum_{i=1}^n [\log P(w_i | t_i) + \log P(t_i | t_{i-1})]$$

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$$40^{20} = 109,951,162,777,600,000,000,000,000,000$$

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Is there a better way?

Dynamic programming: Viterbi

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- ...but I only compute it once!

$$P(t_i|t_{i-1})$$

Example: $P(\text{VB}|\text{MD}) = 0.7968$

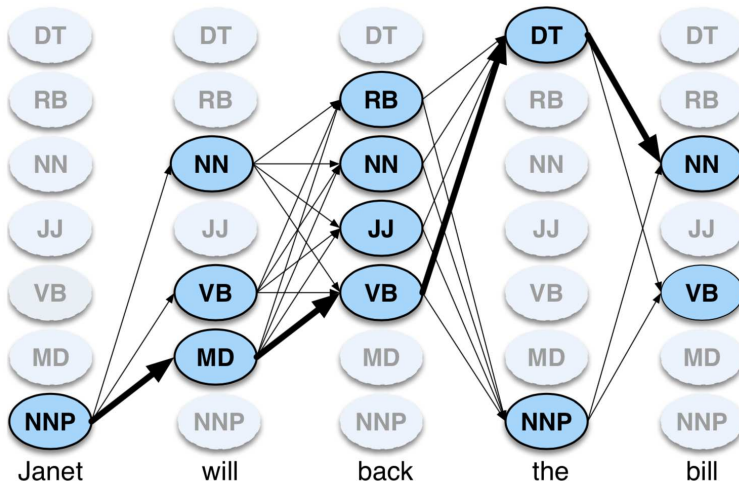
	NNP	MD	VB	JJ	NN	RB	DT
$\langle s \rangle$	0.2767	0.0006	0.0031	0.0453	0.0449	0.0510	0.2026
NNP	0.3777	0.0110	0.0009	0.0084	0.0584	0.0090	0.0025
MD	0.0008	0.0002	0.7968	0.0005	0.0008	0.1698	0.0041
VB	0.0322	0.0005	0.0050	0.0837	0.0615	0.0514	0.2231
JJ	0.0366	0.0004	0.0001	0.0733	0.4509	0.0036	0.0036
NN	0.0096	0.0176	0.0014	0.0086	0.1216	0.0177	0.0068
RB	0.0068	0.0102	0.1011	0.1012	0.0120	0.0728	0.0479
DT	0.1147	0.0021	0.0002	0.2157	0.4744	0.0102	0.0017

$P(w|t)$

Example: $P(the|DT) = 0.506099$

	Janet	will	back	the	bill
NNP	0.000032	0	0	0.000048	0
MD	0	0.308431	0	0	0
VB	0	0.000028	0.000672	0	0.000028
JJ	0	0	0.000340	0	0
NN	0	0.000200	0.000223	0	0.002337
RB	0	0	0.010446	0	0
DT	0	0	0	0.506099	0

Key idea of Viterbi: Lattice



function VITERBI(*observations* of len T , *state-graph* of len N) **returns** *best-path*, *path-prob*

create a path probability matrix *viterbi*[N, T]

for each state s **from** 1 **to** N **do** ; initialization step

$viterbi[s, 1] \leftarrow \pi_s * b_s(o_1)$

$backpointer[s, 1] \leftarrow 0$

for each time step t **from** 2 **to** T **do** ; recursion step

for each state s **from** 1 **to** N **do**

$viterbi[s, t] \leftarrow \max_{s'=1}^N viterbi[s', t-1] * a_{s', s} * b_s(o_t)$

$backpointer[s, t] \leftarrow \arg\max_{s'=1}^N viterbi[s', t-1] * a_{s', s} * b_s(o_t)$

$bestpathprob \leftarrow \max_{s=1}^N viterbi[s, T]$; termination step

$bestpathpointer \leftarrow \arg\max_{s=1}^N viterbi[s, T]$; termination step

$bestpath \leftarrow$ the path starting at state $bestpathpointer$, that follows $backpointer[]$ to states back in time

return $bestpath$, $bestpathprob$

$$P(t_i | t_{i-1})$$

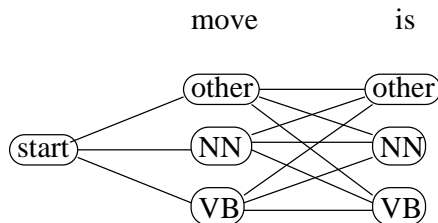
Example: $P(\text{VB} | \text{NN}) = 0.5$

	next	other	NN	VB
prev				
start		0.3	0.4	0.3
other		0.2	0.2	0.6
NN		0.4	0.1	0.5
VB		0.1	0.8	0.1

$P(w|t)$

Example: $P(bear|NN) = 0.45$

	other	NN	VB
bear	0.1	0.45	0.4
is	0.3	0.05	0.05
on	0.3	0.05	0.05
the	0.2	0.05	0.05
move	0.1	0.4	0.45



Goal: Compute

$$\arg \max_{t_1, t_2} p(t_1, \text{move}, t_2, \text{is}) =$$

$$\arg \max_{t_1, t_2} p(t_1 | \text{start}) p(\text{move} | t_1) p(t_2 | t_1) p(\text{is} | t_2)$$

viterbi = vtrb
backpointer = bptr
lattice = path probability matrix

$vtrb_j(t_i)$ is the probability of [the most probable path from 0 to j that tags word w_j with tag t_i].

$bptr_j(t_i)$ is the tag of w_{j-1} on [the most probable path from 0 to j that tags word w_j with tag t_i].

Initialization: $vtrb_0(start) = 1$

Viterbi probabilities for the tags of the first word

$$\text{vtrb}_1(\text{other}) = \text{vtrb}_0(\text{start}) p(\text{other}|\text{start}) p(\text{move}|\text{other}) = 1.0 * 0.3 * 0.1 = 0.03$$

$$\text{vtrb}_1(\text{NN}) = \text{vtrb}_0(\text{start}) p(\text{NN}|\text{start}) p(\text{move}|\text{NN}) = 1.0 * 0.4 * 0.4 = 0.16$$

$$\text{vtrb}_1(\text{VB}) = \text{vtrb}_0(\text{start}) p(\text{VB}|\text{start}) p(\text{move}|\text{VB}) = 1.0 * 0.3 * 0.45 = 0.135$$

Viterbi probabilities for the tags of the second word (1)

$$\begin{aligned} \text{vtrb}_2(\text{other}) &= \max(\\ &\quad \text{vtrb}_1(\text{other}) p(\text{other}|\text{other}) p(\text{is}|\text{other}) = 0.03 * 0.2 * 0.3 = 0.0018, \\ &\quad \text{vtrb}_1(\text{NN}) p(\text{other}|\text{NN}) p(\text{is}|\text{other}) = 0.16 * 0.4 * 0.3 = 0.0192, \\ &\quad \text{vtrb}_1(\text{VB}) p(\text{other}|\text{VB}) p(\text{is}|\text{other}) = 0.135 * 0.1 * 0.3 = 0.00405 \\ &)= 0.0192 \\ \text{bptr}_2(\text{other}) &= \text{NN} \end{aligned}$$

Viterbi probabilities for the tags of the second word (2)

$$\begin{aligned} \text{vtrb}_2(\text{NN}) = \max(\\ & \text{vtrb}_1(\text{other}) p(\text{NN}|\text{other}) p(\text{is}|\text{NN}) = 0.03 * 0.2 * 0.05 = 0.0003, \\ & \text{vtrb}_1(\text{NN}) p(\text{NN}|\text{NN}) p(\text{is}|\text{NN}) = 0.16 * 0.1 * 0.05 = 0.0008, \\ & \text{vtrb}_1(\text{VB}) p(\text{NN}|\text{VB}) p(\text{is}|\text{NN}) = 0.135 * 0.8 * 0.05 = 0.0054 \\ &) = 0.0054 \\ & \text{bptr}_2(\text{NN}) = \text{VB} \end{aligned}$$

Viterbi probabilities for the tags of the second word (3)

$$\begin{aligned} \text{vtrb}_2(\text{VB}) = \max(\\ & \text{vtrb}_1(\text{other}) \, p(\text{VB}|\text{other}) \, p(\text{is}|\text{VB}) = 0.03 * 0.6 * 0.05 = 0.0009, \\ & \text{vtrb}_1(\text{NN}) \, p(\text{VB}|\text{NN}) \, p(\text{is}|\text{VB}) = 0.16 * 0.5 * 0.05 = 0.004, \\ & \text{vtrb}_1(\text{VB}) \, p(\text{VB}|\text{VB}) \, p(\text{is}|\text{VB}) = 0.135 * 0.1 * 0.05 = 0.000675 \\ &) = 0.004 \\ & \text{bptr}_2(\text{VB}) = \text{NN} \end{aligned}$$

Probability of the most likely path: $0.0192 = \max_t \text{vtrb}_2(t)$

Last tag of the most likely path: $\text{other} = \arg \max_t \text{vtrb}_2(t)$

First tag of the most likely path: $\text{NN} = \text{bptr}_2(\text{other})$

Result:

$\text{NN other} = \arg \max_{t_1 t_2} p(t_1, \text{move}, t_2, \text{is})$

- Part-of-speech tagging, informal definition
- Part-of-speech tagging, formal definition

$$\operatorname{argmax}_{t_{1,n}} \sum_{i=1}^n [\log P(w_i | t_i) + \log P(t_i | t_{i-1})]$$

- Brown part-of-speech tags
- Parameter estimation: Context

$$\hat{P}(t^k | t^j) = \frac{C(t^j t^k)}{C(t^j)}$$

- Parameter estimation: Word bias

$$\hat{P}(w^l | t^j) = \frac{C(w^l : t^j)}{C(t^j)}$$

- Order of a Markov model
- Viterbi